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2pAAa6. Sound concentration caused by curved surfaces
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In room acoustics the focusing effect of reflections from concave surfaces is a well-known problem. The occurrence of concave surfaces has tended to increase in modern architecture, due to new techniques in design, materials and manufacturing. Focusing can cause high sound pressure levels, sound coloration or an echo. Although the problem is well known, the amount of amplification that occurs in the focusing point and the sound field around the focusing point are not. The pressure in the focusing point can only be calculated using wave-based methods. An engineering method that is based on the Kirchhoff Integral is presented to approximate the reflected sound field in and around the focusing point for a few basic geometries. It will be shown that both the amplification and the area of the focusing is strongly related to wavelength. The focusing caused by surfaces that are curved in two directions (sphere, ellipsoid) is much stronger than that caused by surfaces that are curved in only one direction (cylinders). This method enables designers to evaluate and thereby improve or redesign the geometry. The method is illustrated with a few examples. This article is the summary of a recently finished PhD research [1].

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GEOMETRICAL ACOUSTICS

In the case of a sound source in the centre of a sphere, the sound reflected from the sphere will concentrate in the centre of the sphere. Using geometrical acoustics, the sound will concentrate in an infinitely small point, resulting in an infinitely high sound pressure. This example illustrates one limitation of geometrical acoustics. Another, more practical limitation is that computer programs that use mirror image or ray tracing algorithms are usually not capable of using curved surfaces, the surfaces have to be segmented. The calculation result depends on the size of the segments. Another reason for wrong results in the concentration area is that energy is added, in case of phase related contributions pressure should be added. However outside the sound concentration area the problems are much less and the sound pressure can be calculated with a geometrical formula. A practical formula is (see also [2],[3]):

$$\Delta L_c = -10 \log \left( \frac{1}{\left( \frac{1}{u} + \frac{1}{s} \right) R_x \cos \theta_x} - 1 \right) - 10 \log \left( \frac{1}{\left( \frac{1}{u} + \frac{1}{s} \right) R_y \cos \theta_y} - 1 \right)$$

where:
- $\Delta L_c$ amplification of the sound level of the reflection due to the curvature (0 dB for a flat surface) [dB]
- $u, s$ distance between source and receiver to the reflecting surface [m]
- $R_x, y$ radius of curvature of the surface, in x or y direction [m]
- $\theta_x, y$ angle of incidence x or y direction

For cylinder segments only one of both terms are to be applied ($R_y = \infty$).

WAVE FIELD APPROACH

Inside the sound concentration area a wave field approach is necessary. A method to calculate the pressure of a reflection is the Kirchhoff integral. Based on Green’s second theorem it can be derived that for every point A inside a volume, the pressure can be determined from the pressure and particle velocity on the (virtual) surface $S$ of this volume:

$$p(\vec{r}_A) = \frac{1}{4\pi} \int_S \left( \frac{p(\vec{r})}{u} \cos \varphi \frac{e^{-jku}}{u} + j \omega \rho \cdot \vec{v}_u(\vec{r}) \frac{e^{-jku}}{u} \right) dS$$

where:
- $k$ wave number $\omega / c$ and $c$ is the propagation speed of sound in air [m/s]
- $p(\vec{r})$ sound pressure on the surface at the position indicated with vector $\vec{r}$ [Pa]
- $\vec{v}_u(\vec{r})$ particle velocity in the direction of the normal of the surface [m/s]
- $\varphi$ angle between the line between the point on the surface and the receiver point and the normal of the surface

It is assumed that the sound source is outside the volume. For a monopole source the pressure and particle velocity on the surface of the volume can be calculated based on the monopole sound propagation. If the surface is a reflecting surface, the pressure and particle velocity can be calculated from a sound source inside the volume $V$. As a first approximation this pressure and particle velocity, but in the opposite direction, will be taken to calculate the reflection. For the addition of incident and reflected sound wave, this will result in a zero velocity, as should be. Although the calculation is done in frequency domain, so for harmonic signals, the approach is only valid for the first reflection. For multiple reflections the surface is ‘transparent’. The method is verified with measurements on a half ellipsoid, see [1].
The Kirchhoff integral can be solved numerically, e.g. in figure 1 and 6, but the calculation is time consuming. For a few basic geometries (sphere, cylinder, double curved surface) the integral mathematical approximate solutions are derived.

**FIGURE 1.** Calculation example of the pressure of the reflected sound based on (2). Left: half sphere (\( \theta_m = \frac{\pi}{2} \)), right: sphere segment (\( \theta_m = \frac{\pi}{5} \)), \( R=5.4 \text{ m} \), 1000 Hz, area shown: 14 x 21 m., color range: white=+10 dB, black=-20 dB.

**SOUND PRESSURE IN THE FOCUSING POINT**

The reflected sound pressure in the focusing point (centre of the sphere) from a sphere segment with radius \( R \) and opening angle \( \theta_m \) (\( 0 < \theta_m \leq \pi \)), with the source in the centre of the sphere segment is (from (2)):

\[
\Delta L_{1m} = 20 \lg k \left( 1 - \cos \theta_m \right)
\]

where: \( \Delta L_{1m} \) sound pressure level in the focusing point relative to the direct sound at 1 m from the source

For a half sphere \( (\theta_m = \frac{\pi}{2}) \) this will be:

\[
\Delta L_{1m} = 20 \lg k \approx 20 \log f - 35 dB
\]

Note that the SPL of the reflection only depends on frequency, not on the radius of the sphere.

For a cylindrical segment with radius \( R \) the SPL in the focusing point (middle axis of the cylinder):

\[
\Delta L_{1m} = 20 \lg \left( \frac{\theta_m}{\pi} \right) + 10 \lg \left( \frac{\pi k}{R} \right)
\]

This is a fair approximation provided the length of the cylinder \( l \) is at least: \( l > \sqrt{\lambda R} \).

In the case of a full cylinder \( (\theta_m = \pi) \) [4]:

\[
\Delta L_{1m} = 10 \lg \left( \frac{\pi k}{R} \right)
\]

For a half sphere and cylinders with different radius the SPL in the focusing point is shown in figure 2. The focusing effect is quite strong, especially for the sphere.
FIGURE 2. The SPL of the reflection $\Delta L_{1m}$ (relative to the direct SPL at 1m from the source), in the centre of a half sphere (4) and on the centre line of a cylinder with radius $R=4$, $8$, $16$ en $32$ m (6).

If the sound source is outside the centre of the sphere or the cylinder, there is not a full build up of energy in the focusing point. The reduction of SPL in the focusing point $\Delta L_f$ as a function of distance $x$ from the sound source to the centre, can be approximated (for $x < 0.85\sqrt{\lambda R}$) with:

$$\Delta L_f \approx 20 \lg \left( \cos \left( q \frac{x^2}{\lambda R} \right) \right)$$

with: $q = \pi / 2$ for $\theta_m \geq \pi / 2$ and $q = \theta_m$ for $\theta_m < \pi / 2$

For a cylinder the reduction with increasing distance (for $x < 0.75\sqrt{\lambda R}$) can be calculated with:

$$\Delta L_f \approx 20 \lg \left( \cos \left( 1.4q \frac{x^2}{\lambda R} \right) \right)$$

If the distance between source and receiver increases, the maximum amplification due to focusing will slightly decrease. However, for increasing distance between source and receiver, the direct level will be much lower and therefore the relative strength of the (delayed) reflection will increase, resulting in audible echoes.

**SIZE OF THE FOCUSING AREA**

If the size of the focusing area is known, it is possible to determine if relevant receiver positions are within that area or not. For receivers inside the focusing area the wave field approach should be used, for positions outside the focusing are the geometrical approach will do.

Figure 3 illustrates the sound pressure along the axis of a sphere. At a certain distance from the focusing point, a zero sound pressure can be found, due to destructive interference. The first zero can be regarded as the boundary of the focusing area.
FIGURE 3. Example of the calculated sound pressure relative to the pressure in the centre as a function of the relative distance \( z_A/R \) on \( z \)-axis. The geometrical field (\( g \)) and the focusing area (\( w \)) are indicated.

The boundary of the sound focusing area of a sphere can be determined with:

\[
Z_{f1,2} = \frac{R}{\pm \lambda R} \frac{2z_B - R}{s} - W
\]

(9)

where: \( z_B \) is the projection of the source position on the axis of the sphere segment [m]

The distance \( W \) is the distance from the reflecting surface to the focusing point and can be determined from the thin lense formula:

\[
\frac{1}{W} + \frac{1}{s} = \frac{2}{R \cos \theta}
\]

(10)

Normal to the axis, the boundary of the focusing area can be found from:

\[
x_f = \frac{\lambda W}{2R \sin \theta_{wo}}
\]

(11)

The focusing area and the parameters used are illustrated in figure 4.

FIGURE 4. Indication of the focusing area (grey) around the focusing point \( A \), with dimensions \( z_{n,2} \) and \( x_f \).
The size of the focusing area increases for low frequencies. This results in a low frequency audibility of this reflection, although the maximum amplification is lower for low frequencies.

**REDUCTION OF SOUND FOCUSING**

Two obvious methods to decrease the reflected sound energy are sound absorption and sound diffusion.

The reduction, expressed in $\Delta L$, of the reflected energy from sound absorbing materials can be based on the sound absorption coefficient of the material: $\Delta L = 10 \lg (1 - \alpha)$. For practical purposes reductions up to ca. 10 dB can be achieved. Especially for low frequencies these values are hard to obtain.

The reduction, expressed in $\Delta L$, of the reflected energy from an irregular surface can be determined from the scattering coefficient $s$: $\Delta L = 10 \lg (1 - s)$. The scattering coefficient can be determined with a laboratory test method (see [5]), however the amount of available data is limited. From measurements and calculations with the boundary element method (BEM) (see [6]) it is clear that it is hard to realize a scattering coefficient of $\geq 0.9$, especially at low frequencies. This means that a reduction of not more than 10 dB can be achieved.

Comparing this 10 dB to the amplification of the reflected sound from a sphere (see figure 2), it is concluded that the sound amplification is only reduced, not removed. For cylinders the amplification is lower and might be reduced sufficiently, as is proved in practice many times.

An alternative method, to obtain reduction of over 10 dB is the redirection of reflections with angled surfaces. The sound is reflected away from the focusing point. The size of the reflecting surface should be at least $\lambda/2$.

Two cases will be presented to illustrate the effect of focusing.

**ROYAL ALBERT HALL LONDON**

The Royal Albert Hall in London has an elliptical floor plan and an ellipsoid roof construction, see figure 5 and 6. The hall has 500 seats and a volume of approx. 80,000 m$^3$. Right from the opening of the hall, end of the 19th century, a strong echo was audible. This echo is mainly caused by the shape of the roof.

*FIGURE 5. Left: interior of the Royal Albert Hall, London; right: Scale model 1:12 with the flying saucers.*

The construction of the geometrical echo is shown in figure 6. The focusing point is far below floor level, so the sound concentration can be described with geometrical methods. Without the flying saucers, the calculated amplification $\Delta L$ in point M using (1) is approx. 15 dB and the reflection is about 9 dB stronger than the direct sound. A scale model 1:12 of the hall was built at Peutz laboratories in Mook, Netherlands, see figure 5. The measured difference between reflected and direct energy is about 11 dB, which roughly corresponds with the results of the geometrical approximation.
FIGURE 6. Longitudinal section of the Royal Albert Hall with the ellipsoid roof. Positions of the focus points of the ellipsoid are indicated with a blue dot.

The echo in the Albert Hall due to the ellipsoid roof was removed by convex reflectors (the ‘mushrooms’ or ‘flying saucers’). These flying saucers are introduced in 1968 and rearranged based on the scale model research.

The middle part of the ceiling is sound absorbing. As seen from the stage, the view to the upper cove is now fully covered with these flying saucers, leaving a free view of the lower cove. This part of the ceiling has a much smaller radius and does not contribute to the sound concentration, see also figure 7.

FIGURE 7. Plan of the flying saucers in the Royal Albert Hall before (left) and after (middle) renovation. The photo (right) is after renovation.
TONHALLE DÜSSELDORF

The concert hall Tonhalle in Düsseldorf, Germany, is a dome shape concert hall with approx. 1900 seats and a volume of approx. 16,000 m³. Before renovation there was an inner dome made of wooden panels, see figure 8.

FIGURE 8. Interior of the Tonhalle Düsseldorf before (left) and after renovation (right).

These panels were at a slight angle relative to the dome shape. Nevertheless the hall had a very strong echo causing a poor reputation and many orchestras and soloists did not want to perform in this hall any more.

The centre of the dome is slightly above audience level, so for sources on stage, the focusing point is right at ear level in the audience. The calculated amplification $\Delta L_{in}$ of a perfect sphere, using (3), is 13 dB at 500 Hz. For a receiver position at 8 m from the sound source the calculated difference between reflected and direct sound, using (3) and (7), is 25 dB at 500 Hz. However this is for a perfect sphere.

From numerical calculation of the Kirchhoff Integral with the complex geometry of the hall (see figure 9) the calculated difference between reflected and direct sound is approx. 15 dB at 500 Hz, see figure 10. Measurements in the hall before renovation showed a difference of approx. 14 dB at 500 Hz.

FIGURE 9. Visualization of the numerical model of the Tonhalle Düsseldorf.

FIGURE 10. Left: floor plan of the Tonhalle Düsseldorf with indication of the area calculated and source on stage (blue dot); Right: calculated sound pressure level for 3 frequencies, using (2).
The first proposed solution consisted out of convex diffusers behind an acoustically transparent layer, replacing the wooden panels. From scale model research this solution appeared to be insufficient, the remaining reflection was still 10 db above direct sound. The second solution made use of redirecting panels at an angle of approx. 30 degrees relative to the sphere, resulting in a 60 degree reflection angle. Figure 11 shows a cross section of the hall with the concept of these reflectors, the resulting 3d geometry and the 1:12 scale model. This solution proved capable of removing the echo and was implemented in the hall.

**FIGURE 11.** Renovation concept of the Tonhalle Düsseldorf. Left: section with geometrical reflection paths; middle: 3D visualization of the reflector geometry; right: photo of the scale model 1:12.

Figure 8, right, shows the Tonhalle after renovation. Visible is the acoustically transparent wire mesh. Behind this wire mesh are the redirecting reflectors, with blue light on it (from behind the wire mesh). The echo proved to be disappeared. Also other room acoustic improvements were made, such as better audibility on stage and enhanced reverberance. Musicians, audience and critics are very enthusiastic about the acoustic of the hall and the important orchestras and soloists have returned to the hall.

**CONCLUSIONS**

With the method presented here (and in [1]) it is possible to predict sound concentration of the reflection from concave curved surfaces in the design stage. The focusing caused by surfaces that are curved in two directions (sphere, ellipsoid) is much stronger than that caused by surfaces that are curved in only one direction (cylinders). Generally, the possible reduction of the focusing effect that can be achieved by using absorbers or diffusers is not enough to eliminate the focusing effect of double curved surfaces. However, these methods might be sufficient to reduce the focusing caused by single curved surfaces such as cylindrical shapes. If absorption or diffusion is not sufficient, designers can consider either redirecting the reflections or more drastically revising the geometry. It would be preferable to consider the focusing caused by concave surfaces from the early design stages.

**REFERENCES**